The Spectrum of Satellite Breakup and Fragmentation

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Introduction: The objective of this paper is to expose the spectrum of satellite breakup physics and implications for debris production and observables.

Satellite response to the debris environment generally emphasizes small-scale hypervelocity impact or the interaction of intense, coherent radiation with satellite surfaces or internals. There are empirical correlations of fragment size distributions based on arena tests and extremely rare observations of breakups in space. Klinkrad[1] describes well the research on material response to hypervelocity impact such as the ballistic limit[2] for various materials and shielding walls. Smirnov, et. al., report well the phenomenology of breakups of monolithic bodies under the influence of nonuniform internal loading, such as pressurized tanks.[3] They set forth the transformation of elastic energy into fragment kinetic energy. They establish a sound physical framework for bounding the number of fragments. We took advantage of these works in our previous papers. [4]

There is not much research into the response of nonuniform structures to hypervelocity collisions with similarly massive and complex objects. This work generally employs complex hydrodynamic and finite element computation that is not well suited to real time, operational assessment of the consequences of such encounters. We hope to diminish the void between the extremes of microscopic impact and complex hydrocodes.

Our previous reports employed aspects of the framework established by Chobotov and Spencer[5], fundamentally equilibrium, Newtonian approach convolved with fundamentals

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[1] Kilnkrad, H., Space Debris, Models and Risk Analysis, Springer, 2006

[2] The ballistic limit is the velocity required for a particular projectile to reliably penetrate a particular surface.

[3] Smirnov, N.N., Nikitin, V.F., and Kiselev, A.B., Space Debris Production in Different Types of Orbital Breakups, Chapter 7, Space Debris, Hazard Evaluation and Mitigation, N.N. Smirnov, Ed., Taylor and Francis, 2002.

[4] 35. Finkleman, D., Oltrogge, D.L. et al, “Space Debris Birth to Death: Analysis from Concern to Consequences,” Amos 2008 Space Situational Awareness Conference, Maui, HI, 16-19 September 2008

[5] Chobotov, V.A. and Spencer, D.B., Debris Evolution and Lifetime Following an Orbital Breakup, J. Spacecraft, Vol 28, No 6, pp 670 ff, , Nov-Dec 1991.

of the NASA Evolve model.[6] We now explore the spectrum of interactions and debris evolutions possible with realistic combinations of these theories.

The spectrum encompasses Newtonian, semi-elastic energy and momentum transfer through the extreme of little or no momentum exchange and degrees of interaction from virtually all of the mass of the colliders being involved through fractional mass involvement. We observe that the more Newtonian outcomes do not agree well with sparse observations of the few collisions that have occurred in space at high relative velocities.

High speed images of collisions such as the 1962 Nike ICBM intercept and the Delta 183 intercept reveal that the objects appear to pass through each other, emerging as a collection of fragments that then disperse with diverse particular velocities under the influence of gravitation and other astrodynamic forces. We previously introduced the concept of partial involvement in which the portions of the colliders that do not make intimate contact tear off, retaining their parent momenta and subsequently fragmenting as elastic energy is released. We now conjecture that the duration of the collision is so short (fractional milliseconds) that stress waves cannot propagate within the involved portions either, and that the involved masses fragment inertially, each fragment inheriting the velocity of the parent object rather than the involved masses evolving about the vector sum of collider parent masses. We call this “ghosting.” We also observe that ghosted outcomes appear in aggregate to match much more closely observed outcomes, particularly that of the recent Iridium 33 – Cosmos 2251 event.

We will discuss the range of outcomes we predict over the spectrum of interaction and fragmentation models. We will examine how the range of outcomes might affect fragment size, mass, and trajectory evolution, with implications for what might be observable and where the less observable fragments might reside.

Fragmentation Theory

There is a wealth of applicable research in the munitions fragmentation community and condensed matter physics. We are grateful for contributions and advice from prominent members of the munitions fragmentation community, notably Dr. Vladimir Gold, of the Army Research. Engineering, and Development Command, Aberdeen, MD.

Fragmentation is complex and only marginally amenable to fundamental theory. Curran, et. al., wrote an excellent summary of fracture and fragmentation phenomenology[7]. They present theoretical and empirical approaches to the dynamic fracture process, “in which material bonds are broken and voids are created in previously intact material.” Microscopically all materials contain voids and inclusions whose behavior is described by internal state variables in material constitutive relations (stress-strain relationships that

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[6] Johnson, N.L., Krisko, P.H., Liou J-C, Anz-Meador, P.D., NASA's new breakup model of evolve 4.0 , Advances in Space Research Volume 28, Issue 9, 2001, Pages 1377-1384

[7] Curran, D.R., Seaman, L., and Shockey, D.A., Dynamic Failure of Solids, PHYSICS REPORTS (Review Section of Physics Letters) 147, Nos. 5&6 (1987), North Holland, Amsterdam.

encompass elastic modulii and Poisson coefficients). Macroscopically, inherent stresses due to applied forces, such as pressurization, and object architecture introduce length scales into the aggregating voids, inducing cracks, and spreading fracture. We continue to stress that the fragmentation problem has two fundamental length scales that govern fragment mass and velocity distributions.

Fragmentation is dynamic. Disturbances must propagate from their source. The shortest physically possible flaw nucleation time is on the order of a nanosecond. The characteristic time for crack growth and coalescence in metals is on the order of a microsecond. Therefore, there is a wide spectrum of satellite collision fragmentation. Particularly in geosynchronous orbits, satellite conjunction speeds are very small. "Fender benders" are quite different then hypervelocity encounters. This is one more distinction between rapid and leisurely encounters discussed by Alfano.[8] This paper focuses on collisions with relative velocities more than a kilometer/sec. We also concentrate on non-sintered metals, since sintered metals, concrete, and other materials readily decompose into dust. Dust is also probably not a debris hazard.

Summarizing material presented by Curran, et al, and others, fractures propagate at characteristic speeds of millimeters per microsecond, comparable to or less than the relative velocity of colliding satellites. Therefore, the encounter is over before real fragmentation proceeds. Pieces are much more likely to break off in brittle fracture than to bend or tear over time. Subsequently, the strain energy stored at the free surface is released, and the "large" separated pieces disassemble as the disturbance propagates. The particular velocities of the fragments are the result of releasing stored strain energy.

Many experiments and theoretical investigations within the munitions fragmentation community have bounded conclusively the minimum fragment size and spectrum of fragment velocities. Rice, et. al., presented a representative compendium of those results.[9] There is a minimum dynamically produced fragment mass greater than about one grain[10]. Therefore we adopt a minimum debris fragment conservatively at 0.01 grams. According to the references we cite, the mass embodied in smaller particles is absolutely negligible. This greatly diminishes previous estimates of the numbers of fragments associated with satellite collisions. Fragment particular velocities should also be on the order of a km/sec and lower; hence relatively small compared to orbital collision relative velocities of interest. These statements are consistent with Curran's more recent work.[11]

Fragmentation Models

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[8] Alfano, S., Addressing Nonlinear Relative Motion For Spacecraft Collision Probability, 2006 AIAA/AAS Astrodynamics Specialist Conference, August 2006

[9] Rice, D.J., Kreider, W., Garnett, C., and Wilson, L.T., Comparing Fragmentation Characteristics of Tungsten, Tantalum, and Steel, 16th International Symposium on Ballistics Fragmentation, San Francisco, CA, 1996

[10] A grain is 64.799 milligrams.

[11] Curran, D.R. Simple Fragment Size and Shape Distribution Formulae for Explosively Fragmenting Munitions, International. Journal of Impact Engineering, Vol 20, 1997, pp 197-208

We have exercised our experience and meager insight and claim no unique accomplishment. We have extracted what we feel are the most enduring aspects of existing satellite fragmentation work and the thankfully small observational data base. The following is our methodology. Virtually all major approaches can be recovered in some embodiment and choice of independent parameters.

We begin by inverting the well known empirical distributions of numbers of fragments as a function of a characteristic length for explosions and collisions as they are used in Evolve 4 and the ESA MASTER model.[12] For collisions:

$$N_f\left(d \geq l_c\right) = 0.1m^{.75}\left(l_c\right)^{-1.71} \tag{1}$$

where the momentum normalized projectile mass is:

$$m = m_p v_i \cdot 1000 \cdot \frac{\text{sec}}{\text{kg} \cdot \text{meter}} \tag{2}$$

and the specific energy:

$$E_p = m_p \cdot \frac{\left(v_i\right)^2}{2 \cdot m_i} \tag{3}$$

must be less than the threshold for complete disintegration. m_p is the projectile mass, v_i is the relative impact velocity, m_i is the mass of the target object. Similar relationships apply for explosions. The number of fragments greater than a given size (lc) depends on the masses of the two objects and the relative impact velocity in this formulation. Area to mass and additional velocities are inferred from analytical abstractions of NASA correlations which incorporate the ESA Master Model proposed improvements. These are depicted below.

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[12] Klinkrad, H., Space Debris, Models and Risk Analysis, Springer, 2006, eq. 3.32 and 3.33, pp 68-69

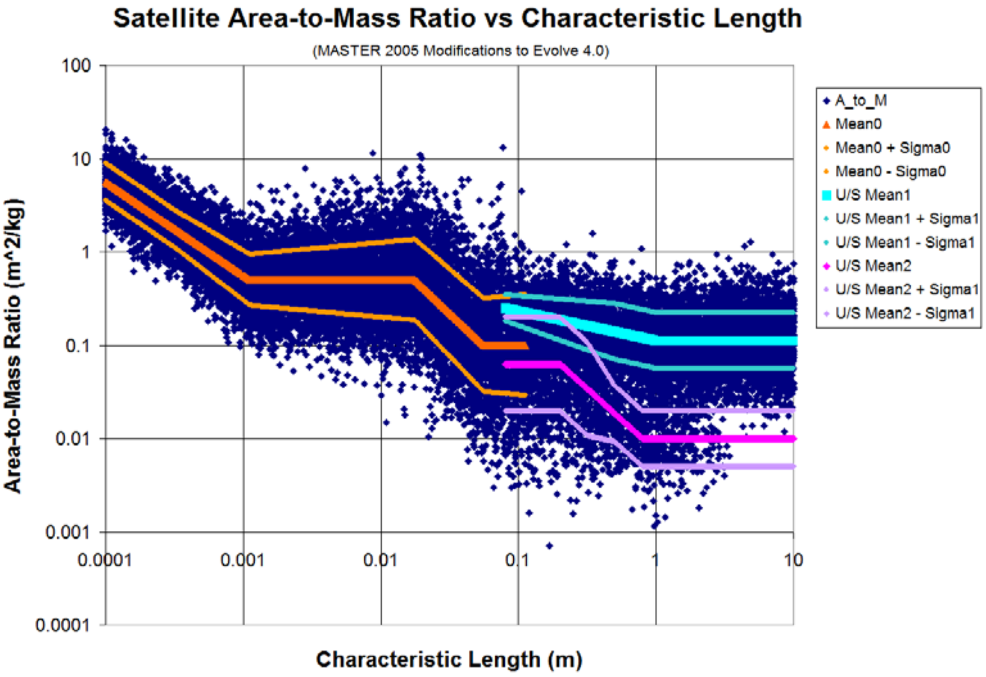

Figure 1: Variation of A/M with characteristic length

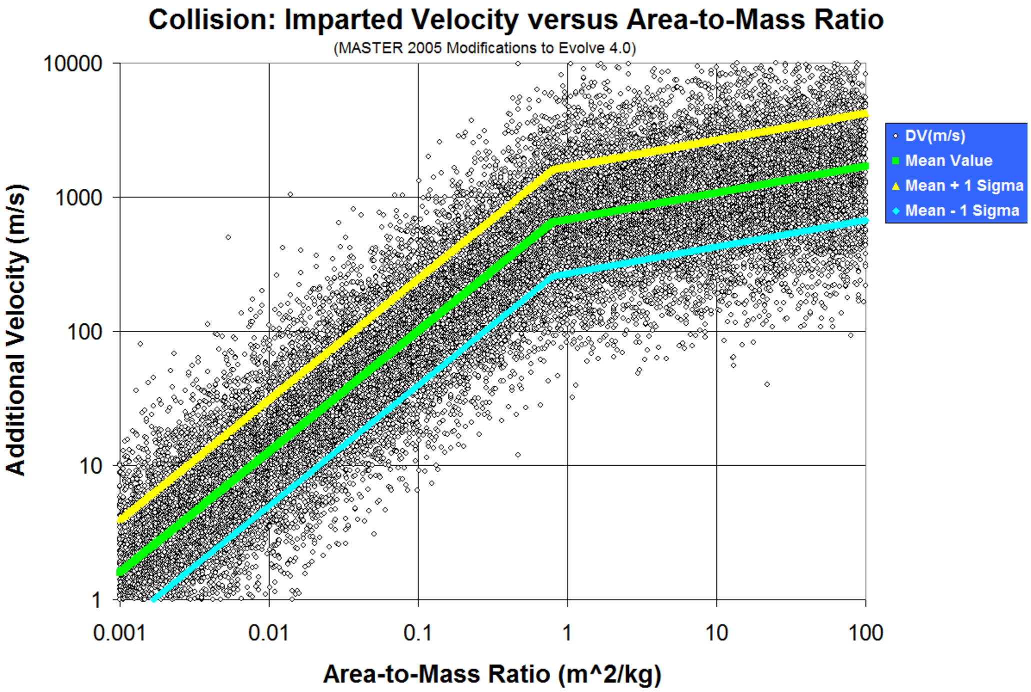

Figure 2: Variation of additional velocity with A/M

These three elements of information yield the number of fragments within a range of characteristic lengths, the area to mass ratios, and additional velocities.

We enter the process with a minimum length scale, as implied by fundamental fragmentation physics, scaled to the characteristic length for Eq 1-3. This leads to numbers of fragments, area to mass ratio, and additional velocity. The trouble is that the total mass of the fragments is almost always not the sum of the initial masses of the colliding objects. Our logic to conserving mass is to reject the most massive fragments until we conserve or accumulate less than the initial total mass. If this undershoots, we scale each fragment mass up to match what we started with.

Then we must conserve momentum. None of the diverse satellite fragmentation models reveals the directions of the additional velocities imparted to fragments. Various assumptions are invoked, such as uniform distribution in all directions relative to the objects' barycenter. Our approach is to draw randomly from vectors distributed in an icosahedron. This distribution has many advantages in data storage, representation, and interpolation, and was first depicted in three dimensions by Leonardo da Vinci. We choose individual fragment additional velocity directions from within this distribution framework so that linear relative momentum is conserved. This is at least based on representative physics.

Finally, energy must be conserved. There is clearly no guarantee, and we must define conservation of energy more concretely. Our technique encompasses and extends most of the current approaches. Initial energy stored within the colliding objects and their classical kinetic plus potential energy must be conserved. But the post-encounter energy can be manifested in four ways: kinetic and potential energy of the fragment centers-of-mass, fragment dynamics such as spinning or tumbling, trapped and stored strain energy, and internal energy of the materials ultimately manifested as radiation as the fragments equilibrate with their surroundings. The last three forms of energy we treat as an aggregated loss. Often the cumulative kinetic energy of the fragments after mass and momentum conservation is less than the total initial energy of the colliders. We take this as an indication that the rest of the energy has been dissipated into the categories noted. If the total energy is greater than the initial energy, then we scale all additional velocities downward until energy is conserved. Whether the universe behaves this way is arguable, but the fact that in a classical paradigm energy is conserved is not arguable. Not conserving mass, momentum, and energy is unreasonable.

$$(4)\quad KE_{orig} - KE_{cm-translation} - Q_{loss} = \eta\left(KE_{orig} - KE_{cm\_translation}\right) = \sum_{i=1}^{N} m_i\left(\bar{v}_{cm} \cdot \Delta\bar{v}_i\right) + \frac{1}{2}\sum_{i=1}^{N} m_i \Delta v_i^{2}$$

A fraction of the initial mechanical energy is unavailable because of dissipation. This is applied to each object. The velocities in Eq 4 are the barycentric velocity of the two colliders and the particular velocities gained by each fragment.

To this end, we invoke the scheme developed by Spencer, Chobotov, and others at the Aerospace Corporation, introducing four fundamental parameters.

η = Initial mechanical energy preserved as mechanical energy among fragments
γ = Fraction of mass of an object involved directly in the collision
ε = Fraction of an object's momentum retained by uninvolved mass
λ = Fraction of additional velocity energy transferred to non-involved mass

For example, if all of the mass of an object were directly involved in the collision and there were no dissipation: $\eta=1$, $\gamma=1$, $\varepsilon=0$, and $\lambda=0$. In practice, the equations have singularities at these precise values, but the approach to them is consistent with reality. Our model has seven putatively independent parameters. In the next section we will explore how varying these parameters affects predicted outcomes.

There are two approaches to "conserving" energy. If we audit kinetic energy after satisfying conservation of mass and momentum, we may find that all of the energy has not been accounted for. We can assume that the deficit is encompassed in dissipation, including the dynamics of the individual fragments. Alternatively, we can impose the condition that initial kinetic and stored energy must be present afterwards and assign a fraction of that energy to dissipation as well as between involved and uninvolved portions of the outcome. In the second case, the number of fragments will be different depending on the degree of dissipation.

Parametric Analysis

Although most analyses invoke Newtonian conservation of momentum and energy, we find this unrealistic and inconsistent with the few observations available. We demonstrate this with a putative conjunction between Iridium 5 and Cosmos 858 early in August 2009. These objects have comparable masses and near head on conjunction geometry. If the collision were Newtonian, post-collision fragment momentum along track would be small, and many fragments would reenter quickly. Figure 1 demonstrates that behavior:

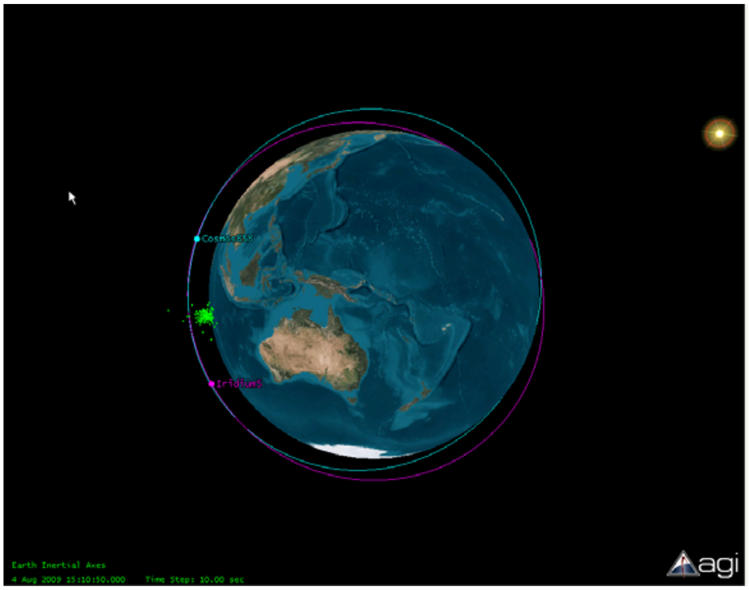

Figure 1: Newtonian behavior for fully involved nearly equal masses and nearly head on conjunction. (Iridium 5, 860 kg; Cosmos 858 750 kg)

This is demonstrably not what happens. Therefore, we examine only "ghosted" conjunctions in the remainder of this paper. Figure 2 is the same conjunction, masses fully involved, but ghosted.

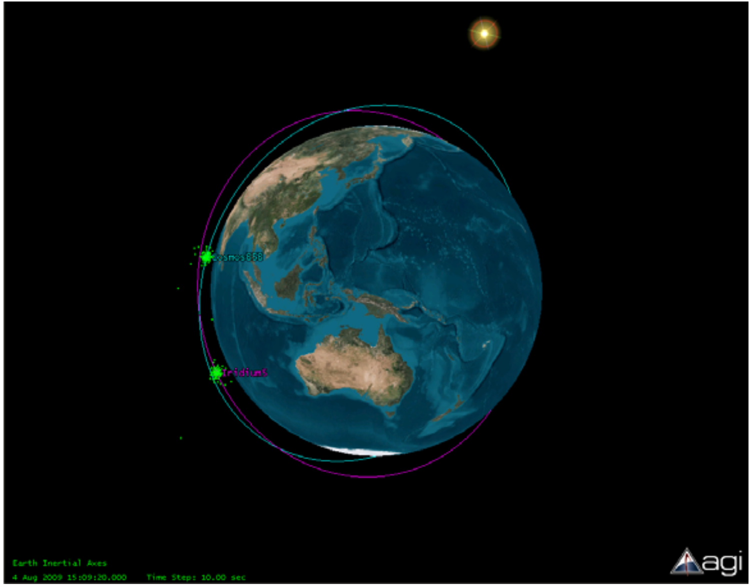

Figure 2: Ghosted conjunction between Iridium 5 and Cosmos 858 with masses fully involved.

Figure 3 illustrates the spectrum of outcomes possible with our comprehensive model depicting the same collision ghosted but with only 50% mass involvement, forced conservation of initial energy, and 20% dissipation.

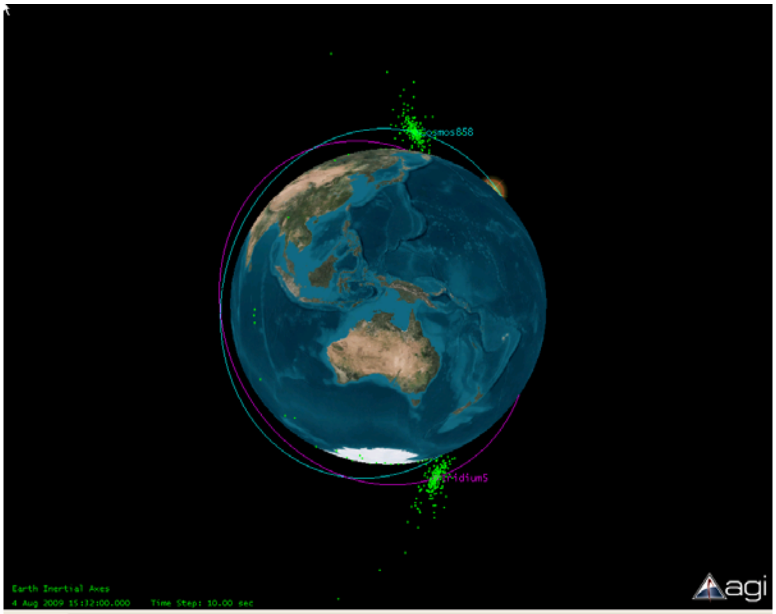

Figure 3: Iridium 5 – Cosmos 858 collision, "ghosted," 50% mass involvement, forced total energy conservation, and 20% dissipation.

These outcomes are quite different including predicting different fragment populations and masses. There is as yet not sufficient forensic information to determine well which among the multiple families of parameters might fit best. We feel it likely that different parameter sets will match different conjunctions since satellites are not of uniform size, material, or design.

Figure 4 shows some of this diversity, comparing perigee altitude and apogee of fragment orbits for two different levels of dissipation.

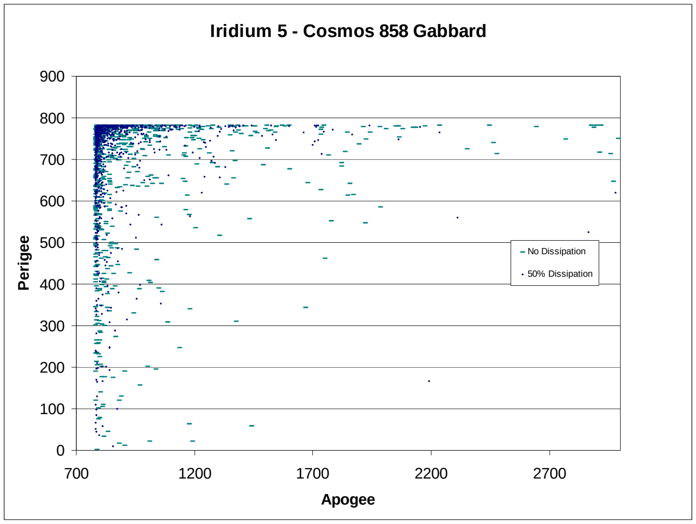

Figure 4: Perigee vs Period (modified Gabbard Plot) for Iridium 5 – Cosmos 858 conjunction with different degrees of dissipation.

The best we can say is that intuitive physics is satisfied. Dissipation causes more fragments to reenter more quickly. Largest fragments achieve higher, more long-lived orbits without dissipation.

Figure 5 shows the velocities imparted to fragments with different levels of dissipation and 50% mass involvement.

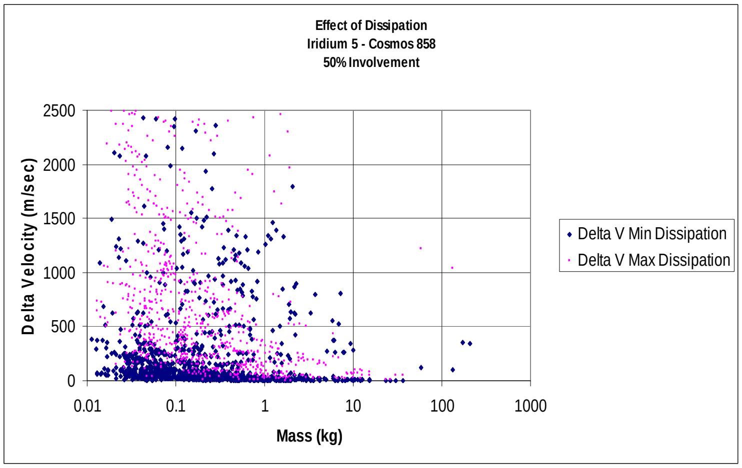

Figure 5: Fragment mass and acquired velocity distributions for the Iridium 5 – Cosmos 858 conjunction with 50% mass involvement.

With dissipation, delta V's are naturally smaller, and the mass distribution includes heavy, slow moving objects. Note that the scale is in meters/second, not kilometers per second.

Figure 6 illustrates the wide spectrum of outcomes that is possible. In this case, we assume that only 10% of the mass of each collision partner is involved and explore the significance of different levels of dissipation. We use the terms "max" and "min" dissipation because 100% and 0% are singularities in our implementation that we have yet to trap. These are effectively with all additional fragmentation energy dissipated and none of it dissipated with momentum and energy assigned to involved and uninvolved portions in proportion to the degree of involvement.

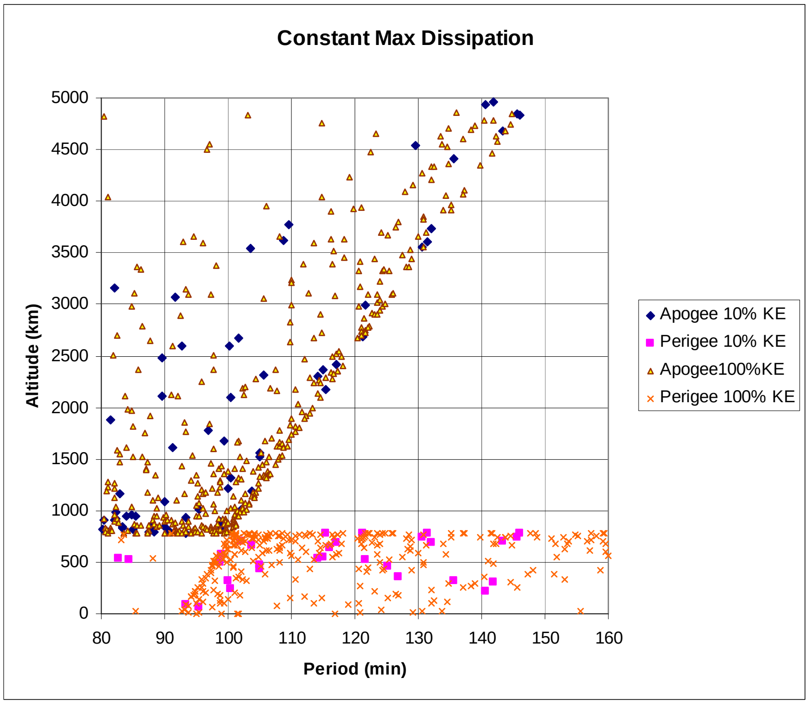

Figure 6: Iridium 5 – Cosmos 858 conjunction Gabbard Plots for 10% mass involvement and different degrees of dissipation.

Interpretation is left to the reader.

Conclusion: We have developed a comprehensive model of the near term consequences of satellite collisions. Our approach embodies most of the theories and correlations that currently exist but recognizes that many characteristics of conjunction geometry, satellite architecture, and physical phenomena can change the outcome. It is fortunate for the space enterprise that there is not sufficient data to narrow the spectrum much. It is unfortunate that we cannot narrow the spectrum much. Our approach is to bound the consequences with reasonable assumptions and try to deal with the worst reasonable outcomes.