Detection of Unknown LEO Satellites Using Radar Measurements

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In the course of processing of radar data with the aim of satellite catalog maintenance a share of measurements does not correlate with cataloged and tracked satellites. These measurements can be used for detection (primary orbit determination) of new unknown objects. The work briefly describes the theoretical foundations and algorithm for solving this task. The characteristics of the algorithm are investigated using mathematical simulation. The efficiency of the suggested method (regarding characteristics of time efficiency and reliability of detected orbits) is demonstrated using the case of satellite break-up in near-circular orbit with insignificant atmospheric drag.

1. INTRODUCTION

The task of detecting of unknown satellites is a part of the general task of satellite catalog maintenance. The theoretical foundations and the description of the algorithm for solving this task used by Russian Space Surveillance System (SSS) are presented in [1, 2]. Some practical results are presented in [3]. However, the orbit detection (primary orbit determination) problems are still a subject of interest for the specialists dealing with processing of significant data fluxes in real time. The recent collision of Kosmos-2251 and Iridium-33 spacecrafts which generated more than 1300 fragments observed by US and Russian space surveillance networks gave additional impulse of interest to this problem. This paper presents certain plausible considerations forming the basis for more strict foundations for the algorithm used in Russian SSS and gives the brief description of the algorithm itself.

The break-ups of tracked satellites are the most severe challenge for orbit detection procedures which most explicitly reveal their features. The most interesting are the characteristics of time efficiency and the reliability of primary determined orbits. The paper suggests to evaluate these characteristics using mathematical simulation of the processes of satellite break-up and the acquisition of radar measurements produced by the fragments. We describe the algorithm used for simulation of radar measurements of the fragments of disintegrated satellite. The algorithm accounts of the basic factors which influence the characteristics of interest.

The catalog maintenance algorithm includes two major processes permanently interacting with each other: detection and tracking.[1,2] Only the consideration of these processes in their interaction can lead to reliable assessment of the characteristics of detection. We briefly describe the joint detection-and-tracking algorithm used in the simulation procedure.

The work presents the results of the simulation of the operation of the detection-and-tracking algorithm for the break-up of a satellite in near circular orbit with insignificant atmospheric drag with altitude about 800 km. This was the altitude where the collision of Kosmos-2251 and Iridium-33 occurred in February 2009. We demonstrate that the described algorithm has rather high efficiency. It is demonstrated as well that the characteristics of time efficiency and reliability of determined orbits significantly depend on the density of the observed break-up fragments.

2. DETECTION ALGORITHM

2.1 Task setting and the used method

The *measurement* hereafter is the six-dimensional vector of positions and velocities in the radar coordinate frame range, azimuth, elevation angle (RCF), which is the result of "smoothing" of unit measurements* of radar coordinates within one pass of a satellite through the radar field of view (normally not more than 50-100 s).

In the course of processing the radar data for catalog maintenance a share of measurements does not correlate with already cataloged and tracked satellites. These non-correlated measurements participate in the detection (primary orbit determination) of new (not cataloged) satellites. The satellite is considered newly detected in case it is missing in the catalog and the primary orbit determination on the basis of non-correlated measurements provides the accuracy sufficient for reliable correlation of future measurements. We will call this *detection condition.*

The group of non-correlated measurements will be called *complete*, if all the measurements of the group belong to one satellite and the orbit generated on their basis satisfies the detection condition. There is always a temptation to use as the initial orbit of a not known satellite the orbit based on one non-correlated measurement, since one measurement always belong to one satellite. If after incorporating into the catalog as a new satellite this orbit always started the process of stable automatic tracking this simplest algorithm would have been the most efficient solution of the detection problem. This would be the case if the errors of the velocity components of the measurement vector have been less than 1 m/s. However, for Russian and US radars these errors are of 1-3 orders of magnitude greater [4, 5]. Thus the simplest scheme of detection based on one measurement can not be enough efficient. Detection of new satellite requires accumulation of several measurements.

Two measurements with distance of one revolution or more usually provide enough accurate primary orbit determination, sufficient for good quality of selection of other measurements. However, rather often we can hardly say that two measurements belong to one satellite with probability close to 1. After a multi-element launch or satellite break-up this situation is typical, since two measurements on two elements with significant errors in velocity components may "inscribe" into one orbit good enough. Therefore, two measurements often do not constitute a complete group.

Three measurements in different revolutions have much more chances to form a complete group, since connection of three measurements for different revolutions and generated by different satellites into one orbit is a much harder task than doing this for two measurements. Thus it is hardly probable to unite in one orbit three measurements among which the two boundary ones (regarding time reference) are generated by one satellite and the middle one by another. This is due to the fact that the error of determination of the position by the time of the middle measurement, calculated using the orbit based on the boundary ones, for the observations in close latitude arguments is of the order of the error of position determination for the time of boundary measurement. Certainly, we can not state for sure that three measurements generated by different objects can not be bonded into one orbit. If the boundary measurements belong to different elements of a break-up they are often united in one orbit and the measurement on some third element may by chance inscribe into this orbit. However this situation is realistic only for the first phase of a break-up when a part of observed fragments have not separated enough and fly within one "tube". Further this becomes less probable.

Thus, the minimum possible size of the complete group of radar measurements in not less than three. That is why the primary orbit determination algorithm looks for three non-correlated measurements for different revolutions for which we can find the orbit inscribing into them. Inscribing of the measurements into the orbit means that the residuals between them are the evaluations of the errors of the measurements and prediction and thus do not contradict with the models of these errors. In case the triplet of measurements and the orbit inscribing all the measurements of the triplet is generated we make an attempt to inscribe other non-correlated measurements into this orbit. If we do find some, the orbit is updated and the attempt is repeated until no non-correlated measurements inscribes into it. The primary orbit determination algorithm

* acquired during one pulse.

based on exhaustive search of triplets is rather laborious. Therefore only relatively high efficiency can justify its use in practice.

The initial information for decision making is the set of non-correlated measurements $\mathbf{M}_{no}$. Here we can find measurements on:

1. Objects newly arrived in space due to launches, separations, break-ups.
2. Objects previously tracked but lost due to long gap in observational data.
3. Objects new to the space surveillance system which exist in orbit for a long time but have not been observed.
4. Tracked satellites in the case of incompliance of the actual errors of the measurements with the models accepted in the measurement correlation process.
5. Phantom objects generated at the sensor by detection of new orbit based on noise marks.

The measurements arriving in $\mathbf{M}_{no}$ due to first three causes constitute the set $\mathbf{M}_{no}^{+}$. They contain useful information needed for solving the detection task. The last two groups of measurements constitute the set $\mathbf{M}_{no}^{-}$. These measurements form the interfering background for the primary orbit determination process. We are not indifferent to the share of measurements $\mathbf{M}_{no}^{-}$ in $\mathbf{M}_{no}$, since the major characteristics of the algorithm: probability of missing the detection, frequency of false detections, time efficiency depend on it. We will assume that this share is insignificant.

Being part of the catalog maintenance software complex the primary orbit determination code runs periodically in real time scale. Thus the set $\mathbf{M}_{no}$ can be divided into two parts: $\mathbf{M}_{no}^{old}$ and $\mathbf{M}_{no}^{new}$. The set $\mathbf{M}_{no}^{old}$ comprises previously arrived (old) measurements for which the detection procedure have already run and which were not included into the orbits generated previously. The set $\mathbf{M}_{no}^{new}$ comprises the newly arrived measurements which have not yet participated in the detection process. Let for the new measurement $\mathbf{x}_{new}$, which will be called the **reference** one, we decided to run the detection computer code. Then the sequence of data fusion is as follows:

1. preliminary selection of triplets containing the measurement $\mathbf{x}_{new}$;
2. generation of the primary orbit for the selected triplet;
3. selection from $\mathbf{M}_{no}$ the measurements inscribing into generated orbit and updating of this orbit by the selected measurements;
4. check of reliability of the updated orbit.

If in the process of the enumerative search we manage to generate the orbit satisfying the reliability criteria, the search with participation of the measurement $\mathbf{x}_{new}$ as a reference one is finished. The reference measurement $\mathbf{x}_{new}$ is changed by the other new measurement from $\mathbf{M}_{no}^{new}$ and the process of the enumerative search starts again.* Note that in the selection of the new triplets the measurements included into the already generated orbit do not participate. This process keeps running until all the measurements from $\mathbf{M}_{no}^{new}$ are processed.

The measurements of reliable orbits are taken away from $\mathbf{M}_{no}$ and the orbits are forwarded to identification with already cataloged satellites. If no identification takes place the new satellite is incorporated into the catalog, which starts participation in the general tracking process. The newly acquired measurements are correlated with the orbit and the orbit is updated. The further sub-sections briefly describe the basic parts of detection algorithm formulated above.

2.2 Preliminary selection of triplets

The number of measurements simultaneously present in the array $\mathbf{M}_{no}$ is counted by thousands [3]. The search of the triplets of measurements constituting a complete group is not an easy task for such an array, since the number of possible combinations of three measurements including the reference one $\mathbf{x}_{new}$ is $\approx 10^6 \div 10^8$. Thus it is expedient to exclude from the search the measurements surely not belonging to the orbit that generated the measurement $\mathbf{x}_{new}$. For this we use the inclination i, longitude of ascending node Ω, the orbital period T and the time t_Ω of passing the equator at the ascending node which are

*This is done as well in the case, when after the end of the search with $\mathbf{x}_{new}$ we did not manage to generate a reliable orbit with participation of this measurement.

calculated using the measurement with rather small errors and with account of these errors simply changes with time. As a result we leave only the measurements satisfying the conditions:

$$|i_{new}-i|<c_i \quad |T_{new}-T|<c_T \quad |\Omega+(t_{new}-t)\dot{\Omega}-\Omega_{new}|_{mod\ 2\pi}<c_\Omega \quad |t_{\Omega,new}-t_\Omega-\tilde{N}T|<c_t \tag{1}$$

where the parameters with index *new* refers to $\mathbf{x}_{new}$, and without the index - to any measurement from $\mathbf{M}_{no}$. Here $\tilde{N}$ – the number of revolutions between $t_{\Omega,new}$ and t_Ω; c_i, c_Ω, c_T, c_t – selecting strobes, which values are selected experimentally with the purpose to make the probability of excluding the measurement belonging to the same object as the $\mathbf{x}_{new}$, less than 0.1.

The set of remaining measurements will be called *the group of the measurement* $\mathbf{x}_{new}$. Only the measurements of this group participate in the search for triplets. The number of measurements in the group $\mathbf{x}_n$ is significantly smaller than in $\mathbf{M}_{no}$. However, it can reach several hundreds even in the case when no new observable objects arrive in space. Within this group we perform the selection of triplets surely not belonging to one satellite without applying the adequate propagator and minimization of the non-linear functional of the least squares method. For this purpose we use the parameter t_Ω of the measurements of the group. Among the three selected measurements one (the left one) has the earliest reference time t_1, the other (the right one) – the latest t_3, the third (middle one) t_2 The right measurement is always the reference measurement $\mathbf{x}_{new}$. In the process of this preliminary selection we run over three parameters: left measurement, middle measurement, the number of revolutions between the boundary measurements N. The enumerative search of the triplets is arranged in the following way. The left measurements are selected according to increasing reference time, beginning with the most "remote". For the fixed left measurement we select the middle measurements to provide that all the three measurements belong to different revolutions. Finally when the measurements are selected the enumeration of possible values of N is performed. The selected triplet along with the evaluation of the number of revolutions between the boundary measurements is forwarded to the primary orbit determination procedure. If certain three measurements were bond into one orbit with k different values of N (number of revolutions between the boundary measurements) the calculation of the primary orbit is performed k times.

2.3 Calculation of the primary orbit

If the triplet of measurements has passed the preliminary selection we make an attempt to generate a primary orbit. For solving this task we perform minimization of the least square functional $\mathbf{\Phi}(\mathbf{a})$. For the vector $\mathbf{a}$ of orbital parameters we take the seven-dimensional vector constituting the six Lagrange's orbital elements λ, L, p, q, h, k and the ballistic parameter s. The ballistic parameter is the area-to-mass ratio (AMR) matched with the used model of the atmosphere. We consider that the primary orbit is generated if the iterative process converged to the certain point $\hat{\mathbf{a}}$, $\mathbf{\Phi}(\hat{\mathbf{a}}) < c_\Phi$ and the value of the functional in this point is smaller than that of the "competitors". The competitors are the orbits constructed for the same triplet of measurements for different values of N, which have passed preliminary selection. Parameter c_Φ is chosen experimentally.

2.4 Calculation of the updated orbit

Apart from the found three measurements, $\mathbf{M}_{no}$ may contain other measurements inscribing into the generated primary orbit. Therefore the "dredging out" of these measurements from $\mathbf{M}_{no}$ is performed. For this purpose we use the algorithm for preliminary correlation of measurements used in the tracking process. The decision on the correlation of the measurement from $\mathbf{M}_{no}$ to the primary orbit is based on the residuals of its radar parameters with the primary orbit. The specific decision function is determined by the model of the real errors of the measurements. For Russian detection radars this model and the adequate decision function are described in [1, 2]. For all the available measurements (the three initial ones and other correlated) we perform the calculation of the orbital parameters. As distinct from the procedure used for calculation of the primary orbit by three measurements, here:

1. For initial approximation we take the already generated primary orbit.
2. For prediction of motion more accurate algorithm is used (the one used in the tracking process).
3. Parameter s is updated.
4. We perform selection of abnormal components and alien measurements.

The measurement is considered alien in case its residuals with the updated orbit contradict the models of the errors of the measurements and prediction. These measurements are returned to $\mathbf{M}_{no}$. The not alien measurement is considered abnormal in case its residuals for some components exceed the given thresholds. One updating in general yields more reliable and accurate orbit. Thus for this orbit the correlation of measurements from $\mathbf{M}_{no}$ and updating the orbit are repeated. This process comes to the end when we find the set of correlated measurements from $\mathbf{M}_{no}$ empty.

2.5 Reliability check

Further decisions depend on the reliability of the obtained orbit. The reliability criteria must provide further stable tracking of the satellite. The orbit is reliable in case it is obtained and the set of measurements inscribing into it is complete. The orbit is obtained in case the iterative process of minimization of $\mathbf{\Phi}(\mathbf{a})$ used for its calculation converged. The measurement is inscribed into the orbit in case its residuals Δ in coordinate parameters are smaller than the thresholds c_{Δ}. The condition of completeness is satisfied in case that the number of inscribed measurements is not less than c_{obs} and the number of revolutions n_{rev}, for which we have inscribed measurements is not less than c_{rev}. The parameters of the decision function (the thresholds c_{Δ}, c_{obs} and c_{rev}) must provide stable tracking of the satellite in future. They are chosen experimentally.

2.6 Orbit identification

Before making the decision that we have detected a new satellite we should check that the newly obtained orbit is not the orbit of previously cataloged but lost satellite. This is the task for the algorithm of orbit identification. The errors of orbital parameters are several orders of magnitude smaller than the maximum errors of the measurements. Thus we have an opportunity to make good decisions for rather long time intervals. The same factor determines the simple structure of the algorithm. The decision is made on the basis of comparing the residuals $\delta\mathbf{a}$ in orbital parameters with empirically chosen thresholds. For calculation of the residuals we use special long-time prediction of the orbit of the cataloged object to the reference time of the new orbit. To reduce the computation burden we perform rough selection of the cataloged objects which for sure can not be identified with the new orbit, using parameters i, Ω, T. Identification of any new orbit is performed with all the cataloged objects, both tracked and lost. When the decision on identification is made the identified cataloged satellite is returned to the tracking process with renewal of orbital parameters. The orbits which are not identified are included into the catalog as new satellites which further participate in the tracking process.

3. DETECTION AND TRACKING ALGORITHM

The algorithm for catalog maintenance includes two major components: primary orbit determination (detection) and tracking. These are two permanent processes interacting with each other [1, 2]. This is the real situation for the processing of radar measurements. For the case of simulation we must have the same process since we can not determine reliable characteristics of primary orbit determination by modeling of this single process. Both processes should be simulated in their interaction. The tracking process described in [1, 2] account of the model of the real errors of Russian radars and thus is rather sophisticated. In particular, for the correlation procedure the requirement to have acceptable probability of miss leads to implementation of laborious minimax algorithm, the orbit updating uses computation consuming adaptive and robust approaches. In the scope of this work we do not simulate the uncertainties in the statistical description of the errors. Thus the tracking algorithm used in the simulation process has been significantly simplified. However, all the mutual relationships between the tracking and detection processes [1, 2] were retained. Further we briefly describe the tracking process that was used in the simulation procedure and the relations with the described above detection process.

The measurements (in reality and in the course of simulation) are processed by the detection and tracking procedure with the time step dt. First we process the measurements acquired in the course of the interval $(t_0,\, t_0{+}dt)$, Then within the interval $(t_0{+}dt,\, t_0+2dt)$, etc.. The input measurement is correlated with the already determined and tracked satellites. The conditions of preliminary correlation of the measurement have the shape: $|\delta r_v| < c_{r_v}$, $|\delta b_1| < c_{b_1}$, $|\delta b_2| < c_{b_2}$, where δr_v, δb_1, δb_2 - the residuals of coordinate parameters of the measurements with the orbit of the tracked satellite along the direction of the motion and two orthogonal (lateral) directions, $c_{r_v}, c_{b_1}, c_{b_2}$ – the strobes determined experimentally. If the measurement was preliminary correlated to several tracked satellites it is attributed to the satellite with minimum value of functional $f = |\delta r_v| + |\delta b_1| + |\delta b_2|$. Non-correlated measurements are forwarded to the detection process described in the previous section.

In case the measurement is correlated to certain satellite we try to update its orbit. We use the same algorithm that is used for updating the primarily determined orbit by the primary orbit determination algorithm. The resulting orbit is tested for reliability. The criteria in the whole coincides with the criteria described for the primary determination algorithm with one addition: the new measurement should be inscribed into the determined orbit. If the updated orbit is reliable we select the alien and abnormal measurements. The criteria in the whole is similar to the criteria described for the detection algorithm with one addition, however: the decisions are made only for the measurements which have at least one measurement with greater time which is inscribed into the orbit. The alien measurements are correlated to all the tracked satellites excluding the satellite from which they have been just now selected as alien. Further for these measurements we can make the same decisions than for the input measurement i.e. they may be correlated to certain cataloged satellite and participate in orbit updating or otherwise enter the array of non-correlated measurements. Such measurements may follow a rather sophisticated way before they stay with certain satellite or remain uncorrelated.

Assume that we do not have a reliable updated orbit and there are no unexpected changes (maneuvers) of orbital parameters. Then we will have only abnormal or alien measurements as causes of unreliability.* In this case we do not record the orbit and make no new decisions regarding the measurements. In case this situation returns several (a parameter selected experimentally) times one after another we consider the satellite to be *lost*. The lost satellites are excluded from the tracking process by the flag prohibiting correlation of measurements with this satellite. All the measurements are declared alien and are removed from the satellite. The lost satellite has only the orbital parameters with no measurements. The satellite can be recovered in the case a newly detected satellite is identified with it.

4. SIMULATION OF MEASUREMENTS FOR THE CASE OF BREAK-UP

In this section we describe the algorithm for simulating the radar measurements on the fragments of a satellite which suffered a break-up. We understand that accurate simulation of the real break-up situation is a hard task and our simulation procedure is very approximate. Nevertheless we tried to take account of all the basic factors important for efficiency and reliability of the algorithm of primary orbit determination.

The input data for the simulation are:

1. n_r – the number of radars for which the measurements are simulated.
2. λ_m, φ_m, h_m – coordinates of the m-th radar (longitude, latitude, altitude above the Earth surface)† $m = 1, 2, ..., n_r$. The index m of the number of the radar further is omitted for simplicity.
3. A_0 – azimuth of the main radiation direction.
4. d_{min}, d_{max}, ε_{min}, ε_{max}, γ_{min}, γ_{max} – boundaries of the field of view of the radar in range, azimuth, elevation angle in the local radar coordinate frame d, ε, γ .‡
5. Π – energy parameter of the radar.
6. t_0 – time of satellite break-up.
7. $\mathbf{a}=(\lambda, L, p, q, h, k)$ – six-dimensional vector of orbital parameters of the satellite for the time t_0.
8. s – area-to-mass ratio of the satellite (AMR).
9. σ_d, σ_ε, σ_γ – RMS errors of single measurements (marks) of range, azimuth and elevation angle.
10. Δ – time interval between the neighboring marks for each satellite.
11. Δt_{min} – minimum time interval for which the measurement on the satellite can be acquired during one pass.
12. t_{end} – time of finishing the simulation.
13. n – the number of break-up fragments.

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* In this work we did not simulate the abnormal errors of the measurements and thus the unreliability may be generated by alien measurements only.

† Origin of the local coordinate frame for the m-th radar.

‡ $x=d \sin\varepsilon \cos\gamma$ $y=d \sin\varepsilon \sin\gamma$ $z=d \cos\varepsilon$, where x, y, z – the axes of the local coordinate frame: x in the plane of local horizon along the main radiation direction, y normal to this plane, z keeps the right character of the frame.

14. $p(\xi,\,\eta,\,\Delta v,\,s,\,l)$ – break-up model – distribution of parameters of the break-up with respect to parameters $\xi,\,\eta,\,\Delta v,\,s,\,l,$ where $\xi,\,\eta,\,\Delta v$ – azimuth, elevation angle and the module of the separation velocity in the orbital coordinate frame $x_o,\,y_o,\,z_o$ of the disintegrated satellite §, $s,\,l$ – AMR and the average size of the fragment.

The simulation algorithm operates as follows:

1. Using the break-up model calculate the parameters $\xi_i,\,\eta_i,\,\Delta v_i,\,s_i,\,l_i$ of all the fragments $(i = 1, 2, ..., n)$.
2. Determine the state vectors $(\mathbf{r}_i,\,\dot{\mathbf{r}}_i)$ for all the fragments for the time t_0:

$$\mathbf{r}_i = \mathbf{r} \qquad \dot{\mathbf{r}}_i = \dot{\mathbf{r}} + \Delta\mathbf{v}_i' \tag{2}$$

where $\mathbf{r},\,\dot{\mathbf{r}} = (X,\,Y,\,Z,\,\dot{X},\,\dot{Y},\,\dot{Z})$ – the state vector of the parent satellite for the time t_0, calculated from **a**. $\Delta\mathbf{v}_i$ – velocity impulse of the i-th fragment, calculated from $\mathbf{r},\,\dot{\mathbf{r}},\,\xi_i,\,\eta_i,\,\Delta v_i$ in the following way *:

$$\begin{gathered}\Delta\mathbf{v}_{oi} = (\Delta v_i\cos\xi_i\cos\eta_i \quad \Delta v_i\sin\xi_i\cos\eta_i \quad \Delta v_i\sin\eta_i)' \\ \mathbf{r}_e = \mathbf{r}'/r \quad \dot{\mathbf{r}}_e = \dot{\mathbf{r}}'/\dot{r} \quad \mathbf{b}_e = \mathbf{r}_e\times\dot{\mathbf{r}}_e \quad \mathbf{n}_e = \dot{\mathbf{r}}_e\times\mathbf{b}_e \\ \mathbf{E} = (\dot{\mathbf{r}}_e\;\mathbf{b}_e\;\mathbf{n}_e) \qquad \Delta\mathbf{v}_i = \mathbf{E}\cdot\Delta\mathbf{v}_{oi}\end{gathered} \tag{3}$$

3. With the step Δ from t_0 until t_{end} we propagate the parameters of all the fragments of the break-up. For each step the propagated positions and velocities of all the satellites are transferred to radar coordinate frame $d,\,\varepsilon,\,\gamma$ of all the sensors and we check the conditions of being in the sector

$$d_{min} < d < d_{max} \qquad \varepsilon_{min} < \varepsilon < \varepsilon_{max} \qquad \gamma_{min} < \gamma < \gamma_{max} \tag{4}$$

If the satellite is in the sector we check the condition of detection

$$\mathrm{P}\cdot(1000/d)^4\cdot(l^2/0.1) > 1 \tag{5}$$

If the detection condition is satisfied, the point (parameters $d, \varepsilon, \gamma,\,\dot{d},\,\dot{\varepsilon},\,\dot{\gamma},$ reference time t, number of the radar m) is marked.

4. Among the marked points with the same number of the sensor and reference time we retain only the isolated ones. For these points in the domain

$$(d - 5\sigma_d,\,d + 5\sigma_d)\cap(\varepsilon - 5\sigma_\varepsilon,\,\varepsilon + 5\sigma_\varepsilon)\cap(\gamma - 5\sigma_\gamma,\,\gamma + 5\sigma_\gamma) \tag{6}$$

there are no marked points from other fragments.

5. Among all the remaining marked points we retain only the points corresponding to representative penetrations of the break-up fragments. The penetration of certain fragment is considered representative in case the penetration interval is longer than Δt_{min}. It means that for this satellite we have marked not less than $\left[(\Delta t_{min}/\Delta\right] + 1$ point consequently, where [A] – entire part of A, rounded up.
6. Among all the marked points of each penetration we retain only the point within the interval Δt_{min}, For this purpose by turns from each end of the penetration interval we remove one point until the remaining interval becomes shorter than Δt_{min}.
7. For each penetration we select the point corresponding to its middle. Its reference time is mostly close to the middle of the interval where the other points corresponding to the penetration are located. Its parameters $d,\,\varepsilon,\,\gamma,\,\dot{d},\,\dot{\varepsilon},\,\dot{\gamma}$ along with the reference time t are fixed as the precise parameters of the measurement.

   As a result of the fulfilled operations (items 1-7) we have formed the precise parameters of all the measurements and all marks.

8. The precise values of the parameters of the measurements and marks are superimposed with the errors of the measurements. † We consider that the errors of all the measurements are not correlated and have normal distribution with zero mean and given RMS errors (different for different parameters and different sensors). The RMS of the errors of the single measurements are the input values $\sigma_d,\,\sigma_\varepsilon,\,\sigma_\gamma$. RMS

§ Origin in the point of satellite location in orbit, axis x_o along velocity vector, y_o normal to the orbital plane, z_o keeps the frame right-hand.

* $\times$ – is the sign of vector product.

† One of the known techniques used for generating stochastic numbers is used.

of the errors $\tilde{\sigma}_d$, $\tilde{\sigma}_\varepsilon$, $\tilde{\sigma}_\gamma$, $\tilde{\sigma}_{\dot{d}}$, $\tilde{\sigma}_{\dot{\varepsilon}}$, $\tilde{\sigma}_{\dot{\gamma}}$, superimposed with the parameters of measurement d, ε, γ, $\dot{d}$, $\dot{\varepsilon}$, $\dot{\gamma}$, are:

$$\tilde{\sigma}_d = \sigma_d \qquad \tilde{\sigma}_\varepsilon = \sigma_\varepsilon/\sqrt{n_o} \qquad \tilde{\sigma}_\gamma = \sigma_\gamma/\sqrt{n_o}$$
$$\tilde{\sigma}_{\dot{d}}=3.5\sigma_d/h \qquad \tilde{\sigma}_{\dot{\varepsilon}}=3.5\sigma_\varepsilon/h \qquad \tilde{\sigma}_{\dot{\gamma}}=3.5\sigma_\gamma/h, \tag{7}$$

where $n_o = \Delta t_{min}/\Delta$, $h = \sqrt{n_o}\cdot\Delta t_{min}$.

5. RESULTS OF SIMULATION

The following initial data have been used for the simulation:

1. The number of radars: $n_r = 1$.
2. Radar location: $\lambda = 0$, $\varphi = 0.5$, $h = 0$.
3. Energy parameter of the radar: Π=27.
4. Errors of the measurements: the RMS of the uncorrelated errors of the measured parameters $\sigma_d = 0.05km$, $\sigma_\alpha = 0.001$, $\sigma_\beta = 0.001$; no correlated and abnormal errors; normal distribution of the errors;
5. The field of view of the radar has the limits: for the elevation angle by the minimum value $1°$ and maximum $60°$, for the range d respectively 100 km and 7000 km.
6. The rate of performing single measurements of the satellite: Δ=5 s.
7. The time interval within which the measurements are performed:: Δt_{min}=50 s.
8. Time interval of simulation of radar measurements: 30 days.
9. The orbit of the break-up satellite: almost circular with inclination $65°$ and average altitude above the Earth surface 800 km;
10. The number of break-up fragments: $n = 300$;
11. the distribution of the parameters ξ, η, of the directions of relative velocities of the fragments: uniform within the intervals $(0, 2\pi)$ and $(-\pi/2, \pi/2)$ respectively.

We considered two versions of the values of the average size l, the area to mass ratio s and the velocity Δv in the orbital coordinate frame. Without brackets we present the characteristics for the first variant ("significant" dispersion), in brackets - for the second variant ("weak" dispersion). For the "weak" dispersion all the values of l and Δv are 3 times smaller, and the values of s are 3 times greater than for the "significant" one.

Table 1. Characteristics of the distribution of the parameters fragments for "significant" ("weak") dispersion

sat.class	number of fragments	l (m)	s (m^2/kg)	Δv (m/s)
1	1	3.0 (1.00)	0.003 (0.009)	1 (0.3)
2	4	0.5 (0.17)	0.018 (0.054)	6 (2.0)
3	15	0.3 (0.10)	0.030 (0.090)	10 (3.3)
4	80	0.2 (0.07)	0.045 (0.135)	15 (5.0)
5	200	0.1 (0.03)	0.090 (0.270)	30 (10.0)

Parameters of detection and tracking algorithm were selected as follows:

1. Strobes for generating groups of non-correlated measurements:
   c_i=0.6°, c_Ω=0.8°, c_T=3 min, c_t=0.3 min.
2. Strobes for correlation of measurements with tracked satellites (km):
   c_{r_v}=5+86400$\cdot v\cdot(3\cdot\sigma_T\cdot n_{rev}+|\Delta T|\cdot n_{rev}^2)$, c_{b_1}=5, c_{b_2}=5, where
   a. v – the absolute value of the velocity in km/s,
   b. σ_T – RMS of the error of the period calculated from the measurement (days),
   c. ΔT – decline of the period by the revolution for the tracked satellite (days),
   d. n_{rev} – number of revolutions between the times of the measurement and the orbit of tracked satellite.
3. The thresholds for the residual of the measurement with the orbit of the tracked satellite for making decision "alien measurement": $10\sigma_d$, $5\sigma_\varepsilon$, $5\sigma_\gamma$, $5\sigma_{\dot{d}}$ respectively for the components $d, \varepsilon, \gamma, \dot{d}$ of the measurement where σ_d, σ_ε, σ_γ, $\sigma_{\dot{d}}$ – RMS values for the errors of these components.

4. The thresholds for the residual of the component of the measurement with the orbit of tracked satellite for making decision "abnormal component": 3σ, where σ – RMS value of the error of this component.
5. The strobes for making the decision on the reliability of the orbit generated by the detection algorithm: $c_{\Delta d}=\max(3\sigma_d,\, 1.0)$, $c_\varepsilon = 3\sigma_\varepsilon$, $c_\gamma = 3\sigma_\gamma$, $c_{obz} = 6\,(7)$, $c_{rev} = 3\,(4)$, if the time interval used for determination of the orbit is less (more) than 2 days.
6. The number of consequent unreliable updates to make the decision that a satellite is "lost" – 2.

The computation time for running the detection and tracking program code for the computer with the rate about 3 GHz, may be comparable to the time interval of simulation. To reduce this time we made some important simplification in the computer implementation of the detection and tracking algorithm. For determination of orbit by the measurements for all the phases (for primary determination and for tracking) the measurements used in the least squares procedure were not the single measurements of range, azimuth and elevation angle but the six-dimensional vectors of radar coordinates and velocities obtained by "smoothing" of single measurements within the time interval $\Delta t_{min}=50\,s$. This simplification reduced the computation time at least 3 times. However, this reduced the "resolution" of the algorithm regarding detection of close fragments and the detection process took some more time.

The simulated measurements on the break-up fragments were fed with time interval dt = 20 minutes. The overall number of measurements for 20 days for the first variant of the break-up was 10530, and for the second variant – 10115. The quality of performance of the detection and tracking algorithm was evaluated using the following basic indicators:

1. The total number of measurements for all fragments – n_{obs}.
2. The number of detected and tracked satellites – $n_{det.SO}$.
3. The number of objects with alien measurements – $n_{SO.alobs}$.
4. The number of alien measurements for tracked objects – $n_{al.obs}$.
5. The number of measurements correlated to tracked satellites – $n_{cor.obs}$,
6. The number of measurements not correlated with tracked satellites – $n_{unc.obs}$,
7. The number of lost objects – $n_{br.tr}$.

Table 2 presents for each of the 5 classes of satellites the average (for all satellites of the class) the values of the following parameters: number of measurements per day (meas/day), the number of measurements used for orbit detection (meas. det.), time used for detection (time interval (in days) within which the measurements participating in the detection are located). The data in the table 2 without brackets (in brackets) refers to the "significant" ("weak") break-up.

Table 2. Average characteristics of detection for satellites of different classes for "significant" ("weak") break-up

sat.class	meas./day	meas.det	time det.
1	6 (6)	6 (6)	0.8 (0.8)
2	5 (5)	6 (7)	0.9 (1.0)
3	3 (3)	8 (10)	2.8 (3.5)
4	2 (2)	8 (10)	3.3 (4.3)
5	1 (1)	7 (10)	4.7 (8.1)

The Appendix presents the figures 1-6. Figs. 1 and 2 illustrate how the parameters n_{obs}, $n_{cor.obs}$ and $n_{unc.obs.}$ change with time for "significant" and "weak" break-ups respectively. Figs.3 and 4 for both variants of break-up presents the temporal change of parameters $n_{unc.obs.}$ and $n_{det.SO}$. Figs. 5 and 6 illustrate the temporal change of parameters $n_{al.obs}$ and $n_{al.obs}/n_{cor.obs}$ – absolute and relative number of measurements which were correlated to alien satellites and remained with them.

We can see the following from the figures.

1. For both variants of the break-up the detection and tracking algorithm has been fed by ≈400 measurements. The number of uncorrelated observations $n_{unc.obs}$ for significant (weak) break-up initially was increasing and by ≈3.0 (≈3.7) days of simulation reached ≈1400 (≈1770). Then the $n_{unc.obs}$ decreased and stabilized at the level of 12 (36) by ≈10 (≈20) days of simulation.
2. All 300 break-up fragments were detected and further stably tracked for the variants of "significant" and "weak" break-ups after 10.5 days and 18.5 days respectively.

3. The number of alien measurements for the objects initially increased and decreased further and with respect to the number of correlated measurements the decrease began from the level of 4%. The number of alien measurements for the objects of the "weak" break-up is always by the order of magnitude greater than for the "significant" break-up.

The detailed analysis shows that all the alien measurements remaining with the objects are acquired in the very beginning of the break-up, when the objects have not separated yet. We can see as well that the residuals with the orbits of alien satellites to which these measurements have been correlated, are * at least for one of the 4 parameters of the measurement $d,\ \varepsilon,\ \gamma\ ,\dot{d}$ greater than the thresholds $3\sigma_d,\ 3\sigma_\varepsilon,\ 3\sigma_\gamma,\ 3\sigma_{\dot{d}}$ and for all the parameters $d,\ \varepsilon,\ \gamma,\ \dot{d}$ are smaller than the thresholds $10\sigma_d,\ 5\sigma_\varepsilon,\ 5\sigma_\gamma\ ,5\sigma_{\dot{d}}$.

One satellite was lost. This happened at the 10th day of simulation for the weak break-up. Part of the measurements of the lost object went to the array of uncorrelated measurements. The satellite was detected again and has been identified with the lost satellite. No such events happened further to this object.

We should make one important note. The more dense is the flux of the break-up fragments the longer is the delay of the detection process. For the significant break-up the number of measurements for the satellite by the time of its detection usually coincided with the threshold value (6 or 7 depending on the time interval of the measurements) or exceeded the threshold by 1 and for the weak break-up with fragments having 3 or less measurements per day the difference reached three. The result was the delay of the detection process.

6. Conclusions

1. The considered detection and tracking algorithm demonstrated rather high resolution and efficiency of detection for the most simple case when the break-up occurs in the almost circular low Earth orbit with insignificant atmospheric drag. However, even for this case there are some unused possibilities for further enhancement of the efficiency. First we should mention:
   a. using of single radar measurements for determination of orbits and testing their reliability;
   b. using the fact that for the time of the break-up all the fragments were in the same point in space.
2. It is expedient to consider in future the more difficult case of the break-up in lower orbits where the atmospheric drag is the major perturbation factor. Here we face additional difficulties since the ballistic characteristics of the fragments (the functions characterizing for each fragment the change of area-to-mass ratio (in the direction of the velocity vector) in time) are not known. The simulation of measurements should take into account that this ratio is not uniform in time.
3. The equally difficult is the case of the break-up in the orbit with altitude more than 700 km, where solar radiation pressure exceeds the atmospheric perturbations. Here we have problems similar to the case of atmospheric drag - the area to mass ratio as function of the direction to the Sun is unknown. In addition for higher orbits regular acquisition of radar measurements is an issue, and without them we can hardly solve the detection and tracking problem.

7. REFERENCES

1. Khutorovsky,Z.N., et al, Low-Perigee Satellite Catalogue Maintenance, *Near-Earth Astronomy (Space Debris)*, Russian Academy of Science, Institute of Astronomy, Moscow, 1998, 34-101 (in Russian).
2. Khutorovsky,Z.N., et al, Catalog Maintenance of Low-Earth-Orbit Satellites: Principles of the Algorithm, *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 6, 745-758, 1999.
3. Khutorovsky,Z.N., Satellite catalog maintenance, *Space research*, Vol. 31, issue 4, 101-114, 1993.
4. *Russia's Arms Catalog*. Vol. 5. Air Defense. Moscow: Military Parade Ltd, 1997, ISBN5-7734-0002-2.
5. Vallado,D.A,. *Fundamentals of Astrodynamics and Applications*, Third Edition, Microcosm Press, 2007.
6. Boikov,V.F., et al, Fast prediction procedures for efficient updating of satellite parameters and time to reentry, *Observations of artificial space bodies*, N 84, part 1, 81-89, Moscow, 1988 (in Russian).
7. Boikov,V.F., et al, Prediction Procedures, Used in Satellite Catalog Maintennance, *Proc. of the Navspacom/Russia Orbit Determination and Prediction Workshop*, Washington D.C., 13-15 July 1994.
8. Boikov,V.F., et al. Prediction Procedures, Used in Satellite Catalog Maintenance, *Journal of Guidance, Control, and Dynamics*, Vol.32, No. 4, 2009.

* In compliance with the set parameters of detection and tracking algorithm

APPENDIX

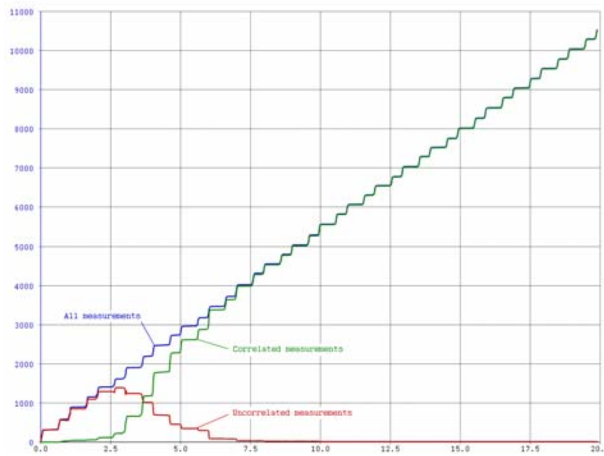

Fig. 1. Parameters n_{obz} , $n_{cor.obz}$, $n_{unc.obz}$ for "significant" break-up as function of time

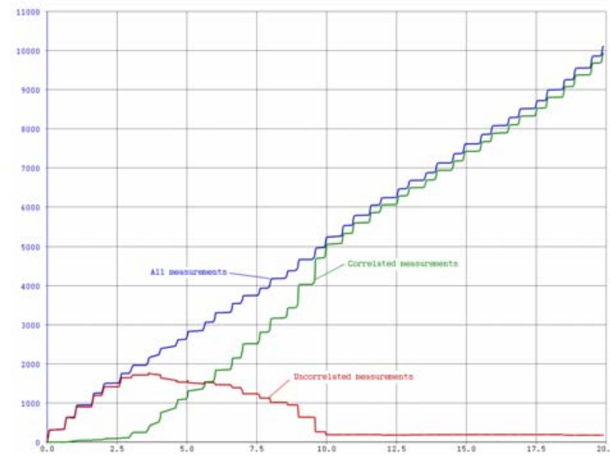

Fig. 2. Parameters n_{obz} , $n_{cor.obz}$, $n_{unc.obz}$ for "weak" break-up as function of time

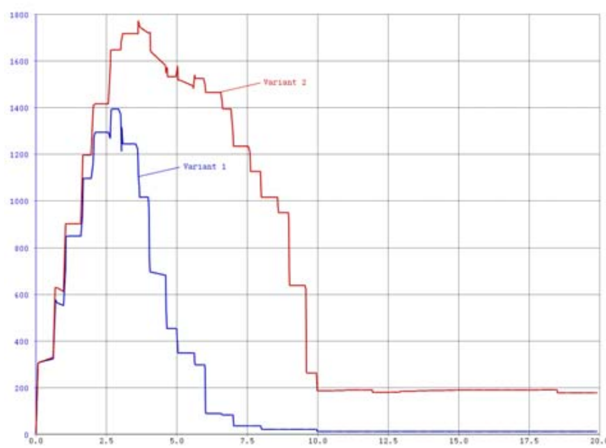

Fig. 3. Parameter $n_{unc.obz}$ as function of time

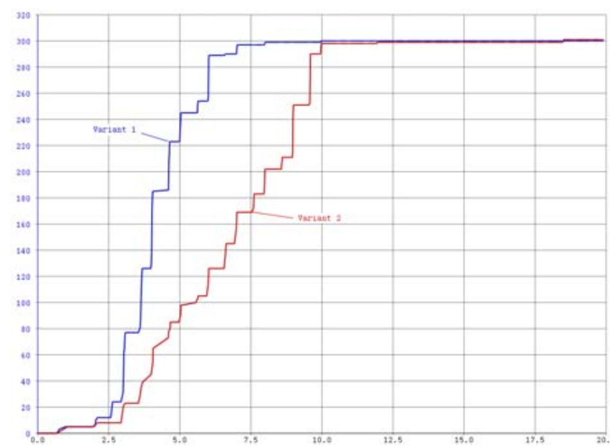

Fig. 4. Parameter $n_{\det.so}$ as function of time

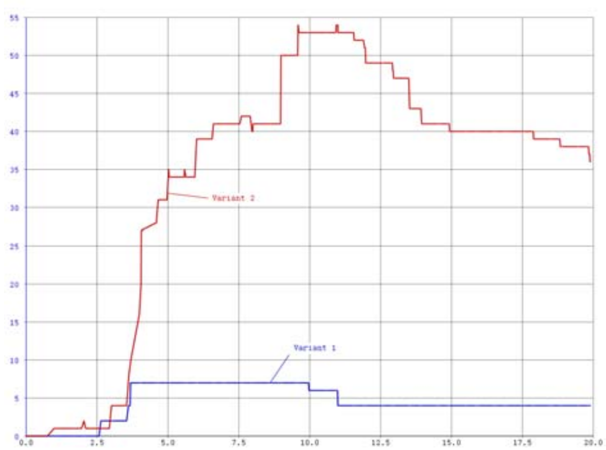

Fig. 5. Parameter $n_{al.obs}$ as function of time

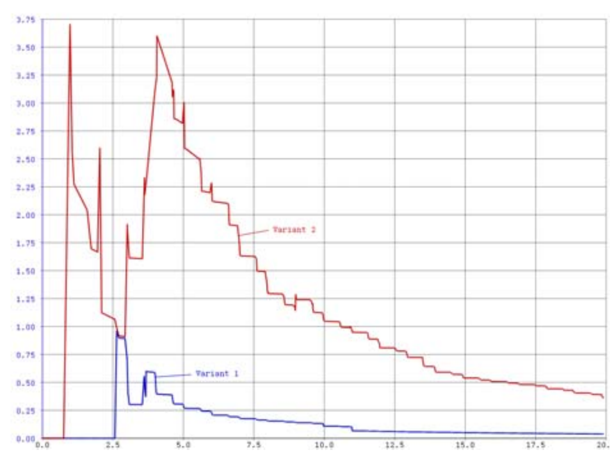

Fig. 6. Parameter $n_{unc.obz}/n_{cor.obz}$ as function of time